Working with Pythagorean Identities

Name **ANSWERS**

Even though there are numerous ways to arrive at the answers for this worksheet, please follow the directions associated with each set of problems. All answers are exact answers unless otherwise stated.

For Questions 1 – 4, use the Pythagorean Identity, $\sin^2\theta + \cos^2\theta = 1$, to support your work. Yes, there are other ways to arrive at the answer, but your task here is to demonstrate how the Identity is involved.

1. If $\cos\theta = \frac{-2}{3}$ and $\tan\theta > 0$, show how to find the value of sine and tangent using a Pythagorean Identity. $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(\frac{-2}{3}\right)^2 = 1;\quad \sin^2\theta + \frac{4}{9} = 1;\quad \sin^2\theta = \frac{5}{9};\quad \sin\theta = \pm\frac{\sqrt{5}}{3}$

$\sin\theta = \frac{-\sqrt{5}}{3};\quad \tan\theta = \frac{\sqrt{5}}{2}$

Tangent is positive in Quadrants I and II, while cosine is negative in Quadrants II and III. θ must be in Quadrant III. Sine is negative in Quadrant III.

2. If $\sin\theta = \frac{3}{5}$ and $\tan\theta = \frac{3}{4}$, show how to find the value of cosine using a Pythagorean Identity.

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{3}{5}\right)^2 + \cos^2\theta = 1;\quad \frac{9}{25} + \cos^2\theta = 1;\quad \cos^2\theta = \frac{16}{25};\quad \cos\theta = \pm\frac{4}{5}$

$\cos\theta = \frac{4}{5}$

Tangent and sine are positive in quadrant I. Cosine is positive in quadrant I.

3. If $\frac{\pi}{2} < \theta < \pi$ and $\sin\theta = \frac{1}{2}$, show how to find the values of cosine and tangent using a Pythagorean Identity. $\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{1}{2}\right)^2 + \cos^2\theta = 1;\quad \frac{1}{4} + \cos^2\theta = 1;\quad \cos^2\theta = \frac{3}{4};\quad \cos\theta = \pm\frac{\sqrt{3}}{2}$

$\cos\theta = \frac{-\sqrt{3}}{2}\qquad \tan\theta = \frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3}$

Cosine and tangent are negative in quadrant II.

4. If $\pi < \theta < \frac{3\pi}{2}$ and $\cos\theta = \frac{-8}{17}$, show how to find the values of sine and tangent using a Pythagorean Identity. $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(\frac{-8}{17}\right)^2 = 1;\quad \sin^2\theta + \frac{64}{289} = 1;\quad \sin^2\theta = \frac{225}{289};\quad \sin\theta = \pm\frac{15}{17}$

$\sin\theta = \frac{-15}{17}\qquad \tan\theta = \frac{15}{8}$

In quadrant III sine is negative and tangent is positive.

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For Questions 5-10, use the Pythagorean Identity, $\sin^2\theta + \cos^2\theta = 1$, to simplify the expressions as directed. More than one solution may be possible.

5. Simplify the expression $(1-\cos^2\theta)(\csc\theta)$ to a single trigonometric function.

$Use: \sin^2\theta+\cos^2\theta=1 \quad \Rightarrow \quad 1\text{-}\cos^2\theta=\sin^2\theta; \quad \text{and} \quad \csc\theta=\dfrac{1}{\sin\theta}$

$(\sin^2\theta)(\csc\theta)=(\sin^2\theta)\left|\dfrac{1}{\sin\theta}\right|=\boxed{\sin\theta}$

6. Simplify $\cos^2\theta+\cos^2\theta\tan^2\theta$.

$Use: \sin^2\theta+\cos^2\theta=1; \quad \text{and} \quad \tan\theta=\dfrac{\sin\theta}{\cos\theta}$

$\cos^2\theta+\cancel{\cos^2\theta}\dfrac{\sin^2\theta}{\cancel{\cos^2\theta}}=\cos^2\theta+\sin^2\theta=\boxed{1}$

7. Simplify the complex fraction at the right into a single trigonometric function. $\dfrac{\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}}{\dfrac{1}{\sin\theta}}$

$\dfrac{\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}}{\dfrac{1}{\sin\theta}} = \dfrac{\dfrac{\sin^2\theta+\cos^2\theta}{\cos\theta\bullet\sin\theta}}{\dfrac{1}{\sin\theta}} = \dfrac{\dfrac{1}{\cos\theta\bullet\sin\theta}}{\dfrac{1}{\sin\theta}} = \dfrac{1}{\cos\theta\bullet\cancel{\sin\theta}}\bullet\dfrac{\cancel{\sin\theta}}{1}=\boxed{\dfrac{1}{\cos\theta}\text{ or }\sec\theta}$

8. Simplify $\sin^4\theta-\cos^4\theta$ into an expression expressed only in terms of sine.

$\sin^4\theta-\cos^4\theta=(\sin^2\theta+\cos^2\theta)(\sin^2\theta-\cos^2\theta)$

$=1\bullet(\sin^2\theta-\cos^2\theta)$

$=\sin^2\theta-(1-\sin^2\theta)$ $\boxed{\sin^2\theta+\cos^2\theta=1 \quad \Rightarrow \quad \cos^2\theta=1-\sin^2\theta}$

$=\sin^2\theta-1+\sin^2\theta=\boxed{2\sin^2\theta-1}$

9. Write the expression $\sec\theta\cos\theta-\cos^2\theta$ as a monomial with a single trigonometric function.

$\sec\theta\cos\theta-\cos^2\theta=\dfrac{1}{\cancel{\cos\theta}}\bullet\cancel{\cos\theta}\text{ - }\cos^2\theta$

$=1-\cos^2\theta=\boxed{\sin^2\theta}$

10. Simplify $\dfrac{\sin^2\theta}{1-\sin^2\theta}$. $\qquad \dfrac{\sin^2\theta}{1-\sin^2\theta}=\dfrac{\sin^2\theta}{\cos^2\theta}=\boxed{\tan^2\theta}$

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